A possible Fermi liquid regime in CuMn and similar spin glass alloys

1991 J. Phys.: Condens. Matter 3 6897

(http://iopscience.iop.org/0953-8984/3/35/019)

J. Phys.: Condens. Matter 3 (1991) 6897–6907. Printed in the UK

A possible Fermi liquid regime in CuMn and similar spin glass alloys

Selman Hershfield
Joseph Henry Laboratories of Physics, Jadwin Hall, Princeton, NJ 08544, USA

Received 4 July 1990, in final form 3 April 1991

Abstract. The very low temperature data for the remanent magnetization, the specific heat, and the resistivity in CuMn and similar glass alloys are examined and it is shown that there is evidence for a Fermi liquid regime. By 'Fermi liquid' it is meant that far below the transition temperature the dominant excitations about a local energy minimum are electronic in character. The spins induce renormalization of electronic properties and interactions between the electrons. Three other possible explanations—spin waves, isolated spins, and droplet excitations—cannot explain all the qualitative features of this data. A second-order, perturbation theory estimate of the Fermi liquid corrections yields results which are too small by up to an order of magnitude.

1. Introduction

Systems of magnetic impurities randomly distributed in a metal have been widely studied both experimentally and theoretically [1–3]. Because the spins interact via the Ruderman, Kittel, Kasuya and Yosida (RKKY) interactions [4] which oscillates rapidly in space, at low temperatures in the more concentrated alloys the spins freeze in random directions, undergoing what is believed to be a spin glass transition [5, 6]. These magnetic impurity alloys and other spin glasses exhibit a rich variety of non-equilibrium phenomena. At low temperatures, however, the non-equilibrium effects become increasingly slow until eventually at sufficiently low temperatures they become negligible on the time scale of most experiments. At such temperatures it is possible to probe the excitations about a state which is a local energy minimum. It is these excitations about local equilibrium which are the focus of this paper.

Because of the random spin–spin interaction and single impurity (Kondo) effects, it is not at all obvious *a priori* what the nature of these excitations is. We thus take a phenomenological approach and first examine the experimental data. In particular we consider the data for the remanent magnetization, the specific heat, and the resistivity for temperatures far below the transition temperature, T_c, defined by the cusp in the AC susceptibility. Just by examining the data we will be able to rule out, or at least seriously question, a number of possible excitations playing a dominant role. The data will also point to an explanation not previously considered in this context, namely that there is a Fermi liquid regime in these alloys. That there could be a Fermi liquid regime is not surprising when one considers other metals in which the impurity degree of freedom is frozen out at low temperatures [7, 8].

0953-8984/91/356897 + 11 $03.50

The outline of the rest of the paper is as follows. First, in section 2 we discuss the relevant experimental results for the remanent magnetization, the resistivity, and the specific heat at low temperatures. The specific heat and magnetization are two of the most fundamental properties of any magnetic system. For a Fermi liquid the resistivity is of equal importance. Next in section 3 we discuss some possible explanations of this data. We begin by showing that the Fermi liquid hypothesis is indeed consistent with all the qualitative features of the data. We then try to go one step further and compute the Fermi liquid corrections from perturbation theory. This calculation elucidates some of the physics of the Fermi liquid corrections, but the results are too small by up to an order of magnitude. Considering the complexity of the system, and that there are no quantitative first-principle calculations of Fermi liquid effects, this discrepancy is not a serious problem. In the remainder of the section we consider three other possible excitations—isolated spins, spin waves, and droplet excitations. All of these have difficulties explaining the data consistently. For example, spin waves cannot account for the anisotropy independence of the remanent magnetization's temperature dependence. Isolated spins cannot explain consistently the temperature dependencies of all three experiments. They are also not consistent with the NMR and direct magnetization measurements in seeing the same magnetization reduction. Droplet excitations should be too large to contribute to the temperature dependence of the resistivity and too slow to account for the magnetization reduction seen by NMR. Thus, it is not easy to find an explanation which is consistent with all the data. In the final section we conclude and list some open theoretical and experimental questions where progress may be possible.

2. Experimental data

2.1. Magnetization

Alloul and coworkers have measured the temperature dependence of the remanent magnetization in CuMn both directly [9] and via NMR [10]. Because the NMR resonance frequency is due to the hyperfine interaction, $\boldsymbol{M} \cdot \boldsymbol{I}$, between the nuclear magnetic moment, $\boldsymbol{I}$, and the electronic magnetic moment, $\boldsymbol{M}$, the shift in the resonance frequency provides a measure of the magnetization reduction, $\delta M = M(T) - M(0)$. In both sets of experiments a large remanent magnetization was used in order to make the effect observable. This remanent magnetization was created by cooling the spin glass in a large field and then turning the field off.

They found that below approximately 0.1 T_c, the decay of the remanent magnetization in time is negligible on the time scale of the experiment. Thus, by cooling down below 0.1 T_c and then back to 0.1 T_c the original remanent magnetization is reproduced. The same behaviour of δM is seen in both experiments.

$$\delta M(T)/M(0) \approx -0.25(T/T_c)^2. \tag{2.1}$$

Besides the temperature dependence, the authors draw a number of other conclusions.

(i) In the NMR experiment the whole distribution of the resonance frequencies shifts, indicating that the reduction takes place fairly uniformly throughout the sample. This is illustrated in figure 1(*a*). Had the reduction in the magnetization been due to a few isolated spins or clusters of spins whose expectation values go from near zero to their maximal value, then the distribution would have gotten wider instead of shifting. This is illustrated in figure 1(*b*). Alloul and Mendels [10] state that it is not possible to account for their data with only 20% of the spins causing the temperature dependence of the magnetization.

(ii) Because the NMR measurements agree with the direct magnetization measurements, the authors claim the magnetization reduction occurs on a time scale which is shorter than the 10^{-9} s of the NMR experiment.

(iii) In the direct measurement of $M(T)$ Alloul *et al* [9] have tested the anisotropy dependence of (2.1) by introducing Pt into their CuMn samples. Transverse susceptibility measurements indicate that the characteristic temperature for the anisotropy, [11] T_A, increases by more than a factor of 20 after the introduction of the Pt. Even though T_A becomes much larger than the temperature, no change in the temperature dependence of (2.1) is observed. Thus, the temperature dependence of the remanent magnetization for these metallic spin glasses seems to be independent or at least very weakly dependent on the anisotropy. In contrast, an insulating Ising spin glass measured in a third set of experiments [12] shows T^3, or even higher, lower-law behaviour at the lowest temperatures measured.

2.2. *Resistivity*

Before examining the temperature dependence of the resistivity far below T_c in these alloys, we recall the general features of $\rho(T)$ at higher temperatures. At the highest temperatures there is a rise in the resistivity as the temperature decreases. This is associated with the Kondo effect [13] modified by the RKKY interaction between the spins. At low temperatures the resistivity decreases with decreasing temperature. This is due to the suppression of spin–flip scattering. The maximum resistivity occurs at around 2 T_c. These two basic points have been discussed by a number of authors starting with Silverstein [14].

The rise and fall of the resistivity is indicative of the two competing energy scales in these alloys: the spin–spin interaction, which tends to freeze the spins, and the electron–spin interaction, which tends to screen the spins. The spin–spin interaction is characterized by T_c, while the electron–spin interaction is characterized by T_K, the Kondo temperature. In these alloys T_c is much greater than T_K. For example, in CuMn the T_cs are around 1 K, while the Kondo temperature is around 1 mK. This separation in energy scales is the reason one usually neglects the electron–spin part of the Hamiltonian in discussing the spin glass state.

A simple way to think about the suppression of the Kondo effect is to say that the local field on a given spin due to the other impurities suppresses the Kondo effect. As shown by experiments on dilute alloys [15], perturbation theory calculations [16], and the exact solution of the Kondo problem [17], an external magnetic field, H, will suppress the Kondo effect. For example, for $T \gg T_K$, $H = 0$ the resistivity satisfies

$$\rho(T, H = 0) = A - B \ln(T) \tag{2.2}$$

while for $H \gg T_K$, $T = 0$ it satisfies [16]

$$\rho(T = 0, H) = A' - B' \ln(H). \tag{2.3}$$

(Unless otherwise noted both the temperature and the fields have units of energy in this paper, i.e. $k_B T \to T$ and $g\mu_B H \to H$.) Within this simple picture a spin glass has a set of local fields, $\{h_i\}$, generated internally by the RKKY interaction, rather than one applied

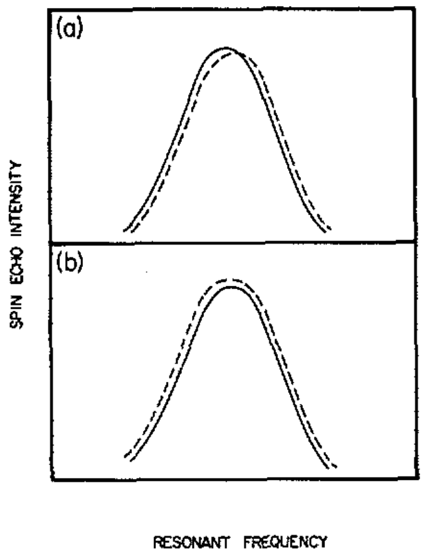

Figure 1. (*a*) The shift in the NMR spectrum seen experimentally, indicating a fairly uniform spin reduction. The broken curve is the lower temperature spectrum. This result is independent of anisotropy. (*b*) The behaviour of the NMR spectrum expected if some small subset of the spins goes from having a near zero expectation value to having the maximal expectation value.

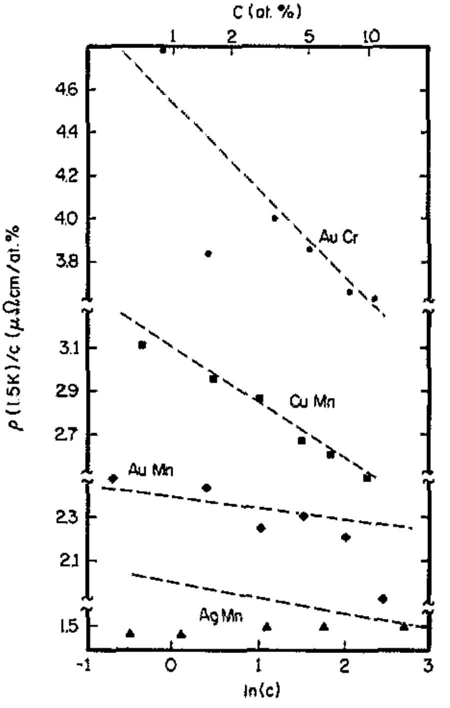

Figure 2. The concentration dependence of the resistivity at 1.5 K for the data of [17]. Each alloy has its own symbol. For these alloys $\rho(1.5\ \mathrm{K}) \approx \rho(0\ \mathrm{K})$. The concentration is measured in atm%. As explained in the text the broken lines are the slopes one would expect from the high temperature Kondo effect in dilute alloys.

field. If the appropriate average local field is proportional to the concentration, c, of magnetic impurities, one would expect to have

$$\rho(T=0) = A'' - B'' \ln(c). \qquad (2.4)$$

In figure 2 we have made four plots of ρ/c versus $\ln(c)$ for the data of Ford and Mydosh [18]. The AgMn sample shows no concentration dependence within the experimental uncertainty, the AuMn and AuCr samples show a trend which could be $\ln(c)$, and the CuMu sample obeys the $\ln(c)$ relation fairly well. The increased scatter in the plots for the lower concentration alloys may be due to there being other sources of the resistivity besides the magnetic impurities which become important at low concentrations. The slopes of the dashed lines in figure 2 come from the perturbation theory relation $B'' = B \times S/(S+1)$ using the Bs from dilute alloys [1]. The heights are adjusted so as to go through the data.

The above, although providing a physical picture for how the Kondo effect is suppressed, is somewhat periphery to our central purpose, namely examining the finite temperature correction to the resistivity, $\delta\rho(T) = \rho(T) - \rho(0)$. Ford and Mydosh [18] found that $\delta\rho(T) \propto T^{3/2}$ at their lowest temperatures; however, for most of their samples

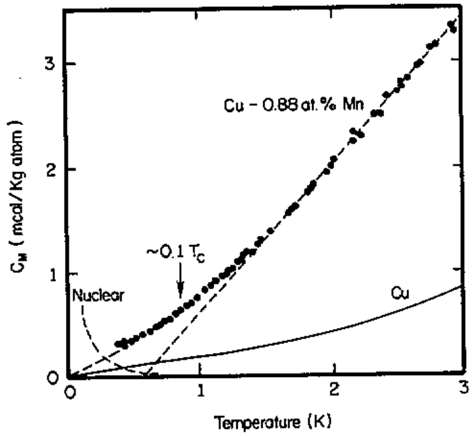

Figure 3. The specific heat of a spin glass alloy. The dots are data taken from Martin [20] for the specific heat of a 0.88 atm% CuMn sample with the specific heat of pure copper subtracted off. The two broken lines going through the data are extrapolations for the first and second linear regimes. The full curve labelled Cu is the specific heat of pure copper, and the broken curve labelled nuclear is the Schottky anomaly in the nuclear specific heat.

they did not get sufficiently far into the very low temperature regime, $T < 0.1\, T_c$, where the remanent magnetization showed a change in behaviour. Lower temperature measurements on CuMn by Laborde and Radhakrishna [19] as well as more recent measurements on CdMn by Albrecht *et al* [20] do find T^2 behaviour. Indeed Ford and Mydosh's high concentration AuCr alloys, which have T_cs around 100 K, seem to show T^2 behaviour at the lowest temperatures. The existence of a $T^{3/2}$ at intermediate temperatures is not necessarily incompatible with a T^2 regime at lower temperatures. Thus, there is some experimental evidence indicating that $\delta\rho(T) \propto T^2$ at the lowest temperatures, although more experimental data will be necessary to show that this behaviour is universal among all the alloys. At present there are no very low temperature measurements ($T < 0.1\, T_c$) of the resistivity in AuFe, AuMn and AgMn. For the CuMn samples of Laborde and Radhakrishna [19] the resistivity satisfies

$$\delta\rho(T)/\rho(T=0) \approx 0.67(T - T_c)^2. \tag{2.5}$$

2.3. *Specific heat*

The dots in figure 3 are data taken from Martin [21] on the low temperature magnetic contribution to the specific heat of a 0.88 at.% CuMn alloy. To get the magnetic contribution to the specific heat, C_M, the specific heat of pure copper and the nuclear specific heat of the Mn have been subtracted from the specific heat of the alloy. (For a Fermi liquid regime C_M should be regarded as an enhancement of the electronic specific heat rather than a separate magnetic contribution.) The nuclear specific heat is due to the same hyperfine interaction responsible for the NMR signal. To see how well such a subtraction can be trusted we have plotted on the same graph the specific heat of pure copper (full curve) and the Schottky anomaly of the nuclear specific heat (broken curve). The specific heat of pure copper is almost linear in this regime, indicating that the electronic contribution is large compared to the phonon contribution. While the copper specific heat is small (but not negligible) compared to the magnetic contribution, the nuclear specific heat can become much larger than the magnetic contribution at low temperatures. This makes it difficult to measure the magnetic contribution to the specific heat at very low temperatures.

Keeping the above in mind we can draw the following conclusions.

(i) The linear regime of the specific heat which starts above T_c does not extrapolate to zero as the temperature goes to zero, but has an intercept at finite temperature. It is thus misleading to say simply that the specific heat of a spin glass is linear.

(ii) At approximately 0.1 T_c for this CuMn sample there appears to be a second linear regime which does extrapolate to zero, although certainly more data is needed to give convincing evidence for this second linear regime.

In particular it would be most useful to look at the specific heat of higher concentration samples. These samples have higher transition temperatures and hence the second linear regime, if it exists, should occur at a higher temperature. The Schottky anomaly, on the other hand, always occurs at the same temperature. Assuming that there is a second linear regime, the magnetic contribution to the specific heat per spin in the second linear regime for the data in figure 3 satisfies

$$C_M(T) \approx 0.37\, k_B(T/T_c). \tag{2.6}$$

Similar data exist for both AuFe [22] and PtMn [23].

3. Explanations of the data

3.1. *Fermi liquid*

A Fermi liquid regime is a regime where the dominant excitations are electronic in character. The role of the spins is to renormalize the electronic properties and to induce interactions between the electrons. To understand how the spins renormalize the electronic properties consider the simple example of a spin S in a local field h interacting with a sea of electrons. The Hamiltonian for our system, H, is the sum of the non-interacting electron–spin Hamiltonian, H_0, and the electron–spin interaction, H'.

$$H_0 = \sum_{k,s} \varepsilon_k a^\dagger_{k,s} a_{k,s} - hS_z \tag{3.1}$$

$$H' = (J/N) \sum_{k,k'} (a^\dagger_{k,\uparrow} a_{k,\downarrow} S_- + a^\dagger_{k,\downarrow} a_{k,\uparrow} S_+). \tag{3.2}$$

The electron–spin coupling constant is J, and N is the number of atoms in the system. A many-body electron state composed from a Slater determinant is denoted by ψ_{el}. If we include the impurity spin but no electron–spin coupling, then for $T \ll h$ the probability that the spin is not in the $S_z = S$ state is exponentially small. Thus, the relevant wavefunctions for the whole system are $\psi_{el} \otimes |S\rangle$, where ψ_{el} has very few electrons further than T above the Fermi surface and very few holes further than T below the Fermi surface. Turning on the electron–spin interaction means that the state, $\psi_{el} \otimes |S\rangle$ evolves into a state which has some of the impurity spin excited state $|S-1\rangle$.

$$\psi_{el} \otimes |S\rangle \rightarrow \psi_{el} \otimes |S\rangle + \sum_{k,\downarrow,\text{occ}} \sum_{k',\uparrow,\text{emp}} w_{kk'} (a^\dagger_{k',\uparrow} a_{k,\downarrow} \psi_{el}) \otimes |S-1\rangle. \tag{3.3}$$

The $w_{kk'}$ can be computed either by perturbation theory or by using the wavefunction in (3.3) to minimize the energy. In either case

$$w_{kk'} = (\sqrt{2S}\, J/N)/(\varepsilon_k - \varepsilon_{k'} - h) \tag{3.4}$$

and the change in the energy is

$$\delta E = N^{-2} \sum_{k,\downarrow,\mathrm{occ}} \sum_{k',\uparrow,\mathrm{emp}} 2SJ^2/(\varepsilon_k - \varepsilon_{k'} - h). \tag{3.5}$$

If we thermal average over all states ψ_{el}, then

$$\langle \delta E \rangle = N^{-2} \sum_{k,k'} f(\varepsilon_k)(1 - f(\varepsilon_{k'}))2SJ^2/(\varepsilon_k - \varepsilon_{k'} - h). \tag{3.6}$$

One can also compute the magnetization reduction from

$$\delta M(T) = -g\mu_{\mathrm{B}} \sum_{k,k'} f(\varepsilon_k)(1 - f(\varepsilon_{k'}))|w_{k,k'}|^2. \tag{3.7}$$

In order to carry out the sums in (3.6) and (3.7) one converts the sums into integrals over energy. Replacing the density of states by the density of states at the Fermi surface and introducing a finite bandwidth cut-off, (3.6) and (3.7) lead to

$$\delta M(T)/-g\mu_{\mathrm{B}}S = -(\pi^2/24)(3J/E_{\mathrm{F}})^2(T/h)^2 \tag{3.8}$$

$$C_{\mathrm{M}}(T) = (S\pi^2/12)(3J/E_{\mathrm{F}})^2(T/h). \tag{3.9}$$

These equations can also be derived via conventional perturbation theory in a manner similar to the treatment of Andrei, Furuya and Lowenstein for the large field limit of the Kondo problem (Appendix C in [17]).

The process which gives the correction to the resistivity is similar to the one for the specific heat and magnetization except that it involves two electrons instead of one. An electron flips the spin from $S_z = S$ to $S_z = S - 1$, and another electron comes in and flips the spin back. In the standard terminology it is an interaction effect rather than a renormalization of the electron properties. Below, consider the initial state to have the electronic states $k_{1\downarrow}$ and $k_{2\uparrow}$ occupied, and $k_{3\downarrow}$ and $k_{4\uparrow}$ empty. Since the probability of having the magnetic impurity in the 'up' position is near unity at low temperatures, we also take the spin to be in the $S_z = S$ state.

$$|i\rangle = |k_{1\downarrow}, k_{2\uparrow}\ \mathrm{occ}; k_{3\downarrow}, k_{4\uparrow}\ \mathrm{emp}; S\rangle. \tag{3.10}$$

The virtual process involves an electron scattering from an occupied state and flipping the spin so the intermediate state must have $k_{1\downarrow}$ empty and $k_{4\uparrow}$ occupied.

$$|m\rangle = |k_{4\uparrow}, k_{2\uparrow}\ \mathrm{occ}; k_{3\downarrow}, k_{1\downarrow}\ \mathrm{emp}; S-1\rangle. \tag{3.11}$$

In the final state the spin S returns to its original position.

$$|f\rangle = |k_{3\downarrow}, k_{4\uparrow}\ \mathrm{occ}; k_{1\downarrow}, k_{2\uparrow}\ \mathrm{emp}; S\rangle. \tag{3.12}$$

The net effect of (3.10)–(3.12) is an effective electron–electron interaction in which the electrons in states $k_{1\downarrow}$ and $k_{2\uparrow}$ are scattered into the states $k_{3\downarrow}$ and $k_{4\uparrow}$. Using second order perturbation theory to compute the amplitude and the 'golden rule' to compute the scattering rate, the resistivity obtained from the Boltzmann equation [7] is

$$\delta\rho(T)/\rho_{\mathrm{S}} = ((\pi^2 + 1)/8)(3J/E_{\mathrm{F}})^2(T/h)^2 \tag{3.13}$$

$$\rho_{\mathrm{S}} = (\pi/2)(m/ne^2)(3J/E_{\mathrm{F}})^2(JS^2/N). \tag{3.14}$$

The ρ_{S} in (3.14) is the resistivity in the first Born approximation.

The above shows how coupling a frozen magnetic impurity to the electron gas allows there to be power law as opposed to exponential temperature dependence of the specific heat, the remanent magnetization, and the resistivity. The form of these temperature dependencies is determined purely by statistics. A spin glass does not have one local field, but a distribution, $P(\mathrm{h})$, of them. In the simple theory one should perform an average over the local fields. The T^p temperature dependencies of (3.8), (3.9) and (3.13) remain unchanged provided that for small h, $P(h) \propto h^q$ with $q > p - 1$. For spins orientated in random directions $q = 2$ because all three components of the local field have smooth distributions as $h_\alpha \rightarrow 0$, $\alpha = x, y, z$ and d^3h equals $4\pi h^2 dh$ (see [24]). To get $q < 2$ would require that the distribution of local fields in the three-dimensional space has a singularity as $h \rightarrow 0$. Thus, on general grounds we have $\delta M \propto -T^2$, $C_M \propto T$, and $\delta\rho \propto T^2$ for a Fermi liquid. All of these are observed experimentally.

There are other qualitative features of the data which fit Fermi liquid behaviour.

(i) The magnetization reduction for a Fermi liquid regime should occur uniformly throughout the sample because all the spins can participate in the virtual processes responsible for the Fermi liquid corrections.

(ii) The spin reduction should occur on electronic time scales, which are fast.

(iii) Finally, the Fermi liquid effects should be insensitive to anisotropy because the spins are already assumed to be frozen for the Fermi liquid effects. We will see in section 3.2–3.4 that it is not easy to obtain a description which is consistent with all the qualitative features of the data.

To make a more quantitative comparison to the experiment we make an approximate fit to the distribution of local fields obtained by Walker and Walstedt [25] in their computer simulations

$$P(h) \approx (4T_c/\pi)h^2/(h^2 + T_c^2)^2. \tag{3.15}$$

Performing the average over local fields yields

$$\delta M(T)/-g\mu_B S = -(\pi^2/24)(3J/E_F)^2(T/T_c)^2 \tag{3.16}$$

$$C_M(T) = (S\pi/6)(3J/E_F)^2(T/T_c) \tag{3.17}$$

$$\delta\rho(T)/\rho_S = ((\pi^2+1)/8)(3J/E_F)^2(T/T_c)^2. \tag{3.18}$$

We use a $J \approx 5$ eV determined by Walker and Walstedt [26]. This J has been corrected to account for the d-wave nature of the scattering in CuMn [27]. Substituting into (3.16)–(3.18) then yields

$$\delta M(T)/-g\mu_B S = -0.019(T/T_c)^2 \tag{3.19}$$

$$C_M(T) = 0.10(T/T_c) \tag{3.20}$$

$$\delta\rho(T)/\rho_S = 0.06(T/T_c)^2 \tag{3.21}$$

while the experimental coefficients for (3.19)–(3.20) are 0.25, 0.37 and >0.67, respectively (see (2.1), (2.6) and (2.5)). Clearly, the estimates are too small by up to a factor of ten. This is not discouraging considering that spin–spin interactions have only been included through the local fields and we have only gone to lowest order (second) in the electron–spin interaction. Fermi liquid corrections are notoriously difficult to calculate from first principles.

3.2. *Isolated spins*

By 'isolated spins' we mean groups of one or a few spins that are purely, by statistical chance, weakly connected to the rest of the system. We have seen in section 2.1 that the whole NMR frequency spectrum shifts to higher energies, indicating that the spin reduction is taking place among all, or at least a large fraction, of the spins. This is the primary evidence against isolated single spins or localized clusters of spins causing the spin reduction at low temperatures.

For single isolated spins one can also show that it is not possible to obtain a consistent distribution of local fields to explain all three experimental properties. If we assume that isolated spins cause the linear temperature dependence of the specific heat, then there must be a finite density of states as the excitation energy goes to zero. The specific heat per spin will go roughly like

$$C_M(T) \sim k_B N(0) T \tag{3.22}$$

where $N(0)$ is the density of states at zero energy. The density of states, $N(\omega)$, is normalized to unity. The finite temperature correction to the resistivity, $\delta\rho$, due to isolated spins should decay like $\exp(-\omega/T)$ for temperatures T that are much less than the excitation energy ω. In the opposite limit, $T \gg \omega$, we expect $\delta\rho$ to go to a constant which is of the order of the resistivity of one spin. Thus, we can estimate

$$\delta\rho(T)/\rho_S \sim \int d\omega\, N(\omega)\, e^{-\omega/T} \sim N(0)T. \tag{3.23}$$

A similar argument also shows that $\delta M(T)/M(0) \sim -N(0)T$. Experimentally, it is found that both $\delta\rho$ and δM are quadratic rather than linear in T.

3.3. Spin waves

In insulating Heisenberg ferromagnets and antiferromagnets the dominant low-temperature excitations are spin waves. Some qualitative characteristics of the data are in agreement with what one would expect for spin waves. The T^2 dependence of the magnetization reduction is consistent with spin waves in an antiferromagnet. It is also conceivable that one might get $\delta\rho(T) \propto T^2$ from spin waves in a spin glass. There are, however, difficulties in explaining other parts of the data with spin waves. First and foremost, if spin waves cause the low-temperature properties, then by introducing anisotropies one should be able to see a change in the temperature dependence from power law behaviour to exponential behaviour. In the experiments of Alloul *et al* [9], an increase in the anisotropy temperature, T_A, so that $T_A \gg T$ did not change the temperature dependence of the remanent magnetization.

Another problem with the spin wave hypothesis is that the magnetic specific heat, C_M, appears to be linear at low temperatures (one would naively expect $C_M \propto T^3$ at low temperatures for spin waves.) This means that there is a finite density of states as the excitation energy, ω, goes to zero. However, the spin wave approximation is unstable for a finite density of states at zero energy. To see this we assume that a magnon of energy ω causes a net spin reduction among all the spins equal to $\delta S(\omega)$. For spin waves in a ferromagnet $\delta S(\omega)$ is one, while for spin waves in an antiferromagnet $\delta S(\omega)$ grows as ω^{-1} as ω goes to zero. At a temperature T the typical spin reduction for one spin is

$$\delta S = \int d\omega\, N(\omega)\delta S(\omega)/(e^{\beta\omega} - 1) \tag{3.24}$$

where $N(\omega)$ is the density of magnon states, again normalized to unity. The integral of (3.24) diverges if $N(\omega)$ goes to a constant as $\omega \to 0$. Thus, in order for C_M to be due to spin waves, the specific heat curve in figure 3 must round off at low temperatures. One may argue that C_M in figure 3 does actually round off and that the linear part is due to an error in subtracting off the electronic specific heat, but such an 'error' is precisely what would happen if there is a Fermi liquid regime.

3.4. Droplet excitations

Droplet excitations are large clusters of spins which can flip. They play an important role in the low frequency long distance behaviour in Ising systems [28, 29]. It is not unreasonable for there to be a finite density of states for droplet excitations down to zero excitation energy. Thus, they can play an important role in the low temperature properties of magnetic systems. Because of their size droplet excitations tend to occur on relatively long time scales compared to microscopic time scales. Alloul and Mendels [10] claim that the fluctuations leading to the magnetization reduction occur on a time scale less than, or of order of, 10^{-9} s of nuclear magnetic resonance. Because of their size it is difficult for large droplet excitations to contribute to the temperature dependence of the resistivity. In order for a droplet to contribute to $\rho(T)$ the electrons must excite it from one state to another. This is exceedingly unlikely because the many electron scattering events needed lead to high powers in J. (This does not mean that droplet excitations cannot contribute to the temporal fluctuations in the resistance [30]. The above two arguments indicate some problems with droplet excitations causing $\delta M(T)$ and $\delta\rho(T)$ in these alloys. These arguments, however, are not as strong as the ones against spin waves and isolated spins. What is really needed is a more quantitative theory of droplet excitations in Heisenberg systems with $1/R^3$ interactions.

4. Conclusions

In this paper we have examined the very low-temperature experimental data on the temperature dependence of the specific heat, the remanent magnetization, and the resistivity of CuMn and similar metallic spin glass alloys, and attempted to explain the data with a number of different excitations. Three very natural ones—isolated spins, droplet excitations, and spin waves—were not consistent with all the experimental observations. The Fermi liquid hypothesis, on the other hand, was consistent with all of the qualitative features of the data; however, simple perturbation theory estimates of the Fermi liquid corrections were consistently too small. Further work is needed to see whether Fermi liquid excitations are responsible for the low-temperature behaviour discussed in this paper.

In particular it would be useful to have data on the low-temperature specific heat of the concentrated alloys so that one could avoid the Schottky anomaly in the specific heat until lower values of T/T_c. This would allow one to determine whether the magnetic contribution to the specific heat goes to zero linearly or as some other power of the temperature. The specific heat of a Fermi liquid must be linear as the temperature goes to zero. It would also be useful to have a systematic study of the $T \ll T_c$ resistivity for these alloys to see if $\delta\rho \propto T^2$ is universal among all the alloys. Both C_M and $\delta\rho$ should be checked for anisotropy dependencies as well as their dependence on J, the electron–spin coupling constant (see (3.16)–(3.18)). On the theoretical side, rather than proceed further in perturbation theory for this very complicated system it would be useful to see if one could formulate a conventional Fermi liquid theory for these alloys; by conventional we mean to express the experimentally observable properties in terms of a few Fermi liquid parameters, which can then be obtained from experiment in more than one way. Finally we note that if the Fermi liquid hypothesis is true, then one should consider the effect of the conduction electrons on the excitations between different energy minima.

Acknowledgments

The author would like to thank P DeVegvar, D S Fisher, C Henley, H R Krishnamurthy and L P Levy for useful discussions. This work was supported by NSF grant DMR-8719523.

References

[1] Rizzuto C 1974 *Rep. Prog. Phys.* **37** 147
[2] Kondo J *Solid State Physics* vol 23, ed F Seitz, D Turnbull and H Ehrenreich (New York: Academic) p 183
[3] Binder K and Young A P 1986 *Rev. Mod. Phys.* **58** 801
[4] Ruderman M A and Kittel C 1954 *Phys. Rev.* **96** 99; Kasuya T 1956 *Prog. Theor. Phys.* **16** 45; Yosida K 1957 *Phys. Rev.* **106** 893
[5] Cannella V and Mydosh J A 1972 *Phys. Rev.* B **6** 4220
[6] Edwards S F and Anderson P W 1975 *J. Phys. F: Met. Phys.* **5** 965
[7] Nozieres P 1974 *J. Low Temp. Phys.* **17** 31
[8] Lee P A, Rice T M, Serene J W, Sham L J and Wilkins J W 1986 *Comment Condens Matter Phys.* **12** 99
[9] Alloul H, Mendels P, Beauvillain P and Chappert C 1986 *Europhys. Lett.* **1** 595
[10] Alloul H and Mendels P 1985 *Phys. Rev. Lett.* **54** 1313
[11] Henley C, Sompolinsky H and Halperin B I 1982 *Phys. Rev.* B **25** 5849
[12] Mendels P, Alloul H and Ribault M 1987 *Europhys. Lett.* **3** 113
[13] Kondo J 1964 *Prog. Theor. Phys.* **32** 37
[14] Silverstein S D 1966 *Phys. Rev. Lett.* **16** 466
[15] Monod P 1967 *Phys. Rev. Lett.* **19** 1113
[16] Beal-Monod M T and Weiner R A 1968 *Phys. Rev.* **170** 552
[17] Andrei N, Furuya K and Lowenstein J H 1983 *Rev. Mod. Phys.* **55** 331
[18] Ford P J and Mydosh J A 1976 *Phys. Rev.* B **14** 2057
[19] Laborde O and Radhakrishna P 1973 *J. Phys. F: Met. Phys.* **3** 1731
[20] Albrecht H, Meierling H, Wassermann E F, Zahres H and Hedgcock F T 1982 *Z. Phys.* B **49** 213
[21] Martin D L 1979 *Phys. Rev.* B **20** 368; 1980 *Phys. Rev.* B **21** 1902
[22] Martin D L 1980 *Phys. Rev.* B **21** 1908
[23] Sato T and Miyako Y 1982 *J. Phys. Soc. Japan* **51** 2143
[24] Held C and Klein M W 1975 *Phys. Rev. Lett.* **35** 1783
[25] Walker L R and Walstedt R E 1980 *Phys. Rev.* B **22** 3816
[26] Walstedt R E and Walker L R 1983 *Phys. Rev.* B **11** 3280
[27] Caroli B 1975 *J. Phys. F: Met. Phys.* **5** 1399
[28] Fisher D S and Huse D A 1988 *Phys. Rev.* B **38** 386
[29] Bray A J and Moore M A 1987 *Glassy Dynamics and Optimization* ed J L van Hemmen and I Morgenstern (Berlin: Springer)
[30] Feng S, Bray A J, Lee P A and Moore M A 1987 *Phys. Rev.* B **36** 5624